Math 162: Calculus IIA
Midterm I
February 20, 2014 8:00-9:15am

NAME (please print legibly):  
Your University ID Number:  
Indicate your instructor with a check in the box:

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Time</th>
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<tbody>
<tr>
<td>Michael Bennett</td>
<td>MWF 1:00-1:50 pm</td>
</tr>
<tr>
<td>Mijia Lai</td>
<td>MWF 10:00-10:50 am</td>
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<tr>
<td>Yoonbok Lee</td>
<td>MWF 11:00-11:50 am</td>
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<tr>
<td>Saul Lubkin</td>
<td>MWF 9:00-9:50 am</td>
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<tr>
<td>Kalyani Madhu</td>
<td>TR 2:00-3:15 pm</td>
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- All electronic devices are strictly forbidden.
- Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.
- Please label and circle your final answers clearly.

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<th>QUESTION</th>
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Trig Identities

- $\sin^2 \theta + \cos^2 \theta = 1$
- $\tan^2 \theta + 1 = \sec^2 \theta$
- $\cot^2 \theta + 1 = \csc^2 \theta$
- $\sin(2\theta) = 2\sin \theta \cos \theta$
- $\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$
- $\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$
- $\sin(a) \cos(b) = \frac{1}{2} [\sin(a - b) + \sin(a + b)]$
- $\sin(a) \sin(b) = \frac{1}{2} [\cos(a - b) - \cos(a + b)]$
- $\cos(a) \cos(b) = \frac{1}{2} [\cos(a - b) + \cos(a + b)]$
1. (12 points) Find the area of the region bounded by the curves $y = \tan(x)$, $y = 2\sin(x)$, $x = -\frac{\pi}{3}$ and $x = \frac{\pi}{3}$.

By symmetry, we just compute half of the area.

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 2\sin x - \tan x \, dx$$

$$= \left[ -2\cos x - \ln|\sec x| \right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

$$= 1 - \ln 2$$

$$A = 2 \left( 1 - \ln 2 \right) = \sqrt{2 - 2\ln 2}$$
2. (12 points)

Find the volume of the solid obtained by revolving the region bounded by $x = y^2$ and $x = 4y$ about the line $y = -2$ using the **cylindrical shell** method. (Note: You won’t receive full credits if you use washer method!)

\[ V = \int_0^4 \pi (y+2) \left( 4y - y^2 \right) \, dy \]

\[ = 2\pi \int_0^4 \frac{4y^2 - y^3}{4} + y - 2y^2 \, dy \]

\[ = 2\pi \left( -\frac{y^4}{4} + \frac{2y^3}{3} + 4y^2 \right) \bigg|_0^4 \]

\[ = \frac{256\pi}{3} \]
3. (12 points)

If a spring requires 24 Joules of work to be stretched from 3 meters beyond its natural length to 5 meters beyond its natural length, how much force is required to hold the spring 6 meters beyond its natural length?

\[ f(x) = kx \quad x \in \text{meters beyond natural length} \]

\[ 24 = \int_3^5 kx \, dx = \left. \frac{kx^2}{2} \right|_3^5 \]

\[ 24 = \left( \frac{25k}{2} \right) - \left( \frac{9k}{2} \right) = 18k \]

\[ 18 = 18k \quad \Rightarrow \quad k = 1 \]

\[ F = 3 \cdot 6 = 18 \, N \]
4. **(18 points)** Evaluate the following definite integrals.

(a) (9pt) \( \int_0^1 \frac{e^x}{1 + e^{2x}} \, dx \)

\[ \text{let } u = e^x \quad du = e^x \, dx \]

\[ = \int \frac{du}{1 + u^2} = \arctan(u) = \arctan(e^x) \bigg|_0^1 \]

\[ = \arctan(e) - \arctan(1) = \frac{\pi}{4} \]

(b) (9pt) \( \int_0^1 x^2 e^{2x} \, dx \)

\[ \text{let } u = x^2 \quad du = 2x \, dx \]

\[ dv = e^{2x} \, dx \quad v = \frac{1}{2} e^{2x} \]

\[ = x^2 \cdot \frac{1}{2} e^{2x} - \int xe^{2x} \, dx \]

\[ u = x \quad du = dx \]

\[ dv = e^{2x} \, dx \quad v = \frac{1}{2} e^{2x} \]

\[ = \frac{1}{2} x^2 e^{2x} - \frac{1}{2} xe^{2x} + \frac{1}{2} e^{2x} \bigg|_0^1 \]

\[ = \frac{1}{2} x^2 e^{2x} - \frac{1}{2} xe^{2x} + \frac{1}{4} e^{2x} \bigg|_0^1 \]
5. (30 points) Evaluate the following indefinite integrals:

(a) (10pt) \( \int \cos^4 x \sin^3 x \, dx \)

Let \( u = \cos x \) \( \Rightarrow \) \( du = -\sin x \, dx \)

\[
= -\int u^4 \sin^2 x \, du
\]

\[
= -\int u^4 (1-u^2) \, du
\]

\[
= \int u^6 - u^4 \, du = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C
\]

(b) (10pt) \( \int \sec^4 x \tan^4 x \, dx \)

Let \( u = \tan x \) \( \Rightarrow \) \( du = \sec^2 x \, dx \)

\[
= \int u^4 \sec^2 x \, du
\]

\[
= \int u^4 (u^2 + 1) \, du = \frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C
\]
(c) \(10\text{pt} \int \frac{dx}{(1-x^2)^{3/2}}\)

\[
\text{let } x = \sin \theta \quad dx = \cos \theta \, d\theta \\
= \int \frac{\cos \theta \, d\theta}{\cos^3 \theta} \\
= \int \csc^2 \theta \, d\theta = -\cot \theta - \cot \theta + C \\
= \int \frac{x}{\sqrt{1-x^2}} + C
\]
6. (16 points) Use partial fractional decomposition to evaluate the integral:

\[
\int \frac{3x + 2}{x^3 + 3x^2 + 2x} \, dx.
\]

\[
\frac{3x+2}{x(x^2+3x+2)} = \frac{3x+2}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}
\]

Multiply by \(x(x+1)(x+2)\) to get

\[
3x+2 = A(x+1)(x+2) + Bx(x+2) + C(x+1)x
\]

\[
= (A+B+C) x^2 + (3A+2B+C) x + 2A
\]

\[
\Rightarrow \begin{cases} 
2A = 2 \\
3A + 2B + C = 3 \\
A + B + C = 0
\end{cases}
\]

\[
\Rightarrow A = 1, \quad B = 1, \quad C = -2
\]

\[
\text{Integral} = \int \frac{1}{x} \, dx + \int \frac{1}{x+1} \, dx - \int \frac{2}{x+2} \, dx
\]

\[
= \ln |x| + \ln |x+1| - 2 \ln |x+2| + C
\]