MTH 162: Calculus IIA
Midterm 1
February 21, 2013

NAME (please print legibly): ______________________________________
Your University ID Number: ________________________________
Indicate your instructor with a check in the box:

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Time</th>
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<tbody>
<tr>
<td>Matthew Creek</td>
<td>MWF 1:00 - 1:50 PM</td>
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<tr>
<td>Mark Herman</td>
<td>MWF 10:00 - 10:50 AM</td>
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<tr>
<td>David Karapetyan</td>
<td>MWF 11:00 - 11:50 AM</td>
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<td>Yoonbok Lee</td>
<td>MWF 9:00 - 9:50 AM</td>
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<tr>
<td>Meg Walters</td>
<td>MW 2:00 - 3:15 PM</td>
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• There are no notes, textbooks, etc. allowed on this exam. The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.

• Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

• Clearly circle or label your final answers.

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<tr>
<th>QUESTION</th>
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1. (20 points)

(a) (2 pts) Let $S$ be the region enclosed by the curves $y = x^2$ and $y = 8 - x^2$. Provide a sketch of this region (find and label the intersection points, and $y$-intercepts).

(b) (6 pts) Set-up, but do not evaluate ONE integral that gives the area of region $S$. 
(c) (6 pts) Consider the solid generated by rotating region $S$ about the $x$ axis. **Set-up, but do not evaluate** ONE integral that gives the volume of the resulting solid.

(d) (6 pts) Consider the solid generated by rotating the region $S$ about the line $x = 4$. **Set-up, but do not evaluate** ONE integral that gives the volume of the resulting solid.
2. (14 points) Consider a tank generated by rotating the curve $y = x^2$ on $-2 \leq x \leq 2$ about the $y$-axis (units are in feet). Suppose the tank is completely filled with a fluid that weighs $\delta \text{ lb/ft}^3$. Find the work done to empty the tank by pumping the water through a pipe 2 ft above the tank. Note: your answer should be written in terms of $\delta$. 

\[ y = x^2 \]
3. **(16 points)** Evaluate the following:

(a) (8 pts) \( \int_{0}^{2} xe^{3x^2+1} dx \)

(b) (8 pts) \( \int \frac{3x^2 + 6}{(x^3 + 6x - 7)^9} dx \)
4. (16 points) Evaluate the following:

(a) (8 pts) \( \int x^2 e^x dx \)

(b) (8 pts) \( \int x^{-3} \ln(x) dx \)
5. **(18 points)** Evaluate the following:

(a) (9 pts) \( \int \tan^2 x \sec^4 x \, dx \).

(b) (9 pts) \( \int_0^{\pi/4} x \sec^2 x \, dx \).
6. (16 points) Evaluate the following indefinite integral. \( \int \frac{x^2 \, dx}{\sqrt{3 - x^2}} \). Note: your answer should be simplified where possible.
Trig Identities

- $\sin^2 \theta + \cos^2 \theta = 1$
- $\tan^2 \theta + 1 = \sec^2 \theta$
- $\cot^2 \theta + 1 = \csc^2 \theta$
- $\sin(2\theta) = 2 \sin \theta \cos \theta$
- $\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$
- $\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$
- $\sin(a + b) = \sin(a) \cos(b) + \cos(a) \sin(b)$
- $\sin(a - b) = \sin(a) \cos(b) - \cos(a) \sin(b)$
- $\cos(a + b) = \cos(a) \cos(b) - \sin(a) \sin(b)$
- $\cos(a - b) = \cos(a) \cos(b) + \sin(a) \sin(b)$
- $\sin(a) \cos(b) = \frac{1}{2}[\sin(a - b) + \sin(a + b)]$
- $\sin(a) \sin(b) = \frac{1}{2}[\cos(a - b) - \cos(a + b)]$
- $\cos(a) \cos(b) = \frac{1}{2}[\cos(a - b) + \cos(a + b)]$