MTH 162: Calculus IIA
Final Exam
May 6, 2013

NAME (please print legibly): ____________________________________________
Your University ID Number: ____________________________________________
Indicate your instructor with a check in the box:

- Matthew Creek  MWF 1:00 - 1:50 PM
- Mark Herman     MWF 10:00 - 10:50 AM
- David Karapetyan MWF 11:00 - 11:50 AM
- Yoonbok Lee     MWF 9:00 - 9:50 AM
- Meg Walters     MW 2:00 - 3:15 PM

- There are no notes, textbooks, etc. allowed on this exam. The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.
- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Clearly circle or label your final answers.

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Part A

1. (16 points)

Evaluate the following integrals

(a) \[ \int x^3 e^{x^2} \, dx \]

(b) \[ \int_{-1}^{0} \frac{1}{x^2 + 2x + 2} \, dx \]
2. (16 points)

Evaluate the following integrals

(a) \[ \int \tan^3(x) \sec(x) \, dx \]

(b) \[ \int (9 - x^2)^{3/2} \, dx \]
3. **(12 points)** Determine whether each of the following sequences converges or diverges. If it converges, compute its limit. If it diverges, state whether it diverges to $+\infty$, $-\infty$, or neither.

(a) \[ \left\{ \frac{9n - 1}{5n + 4} \right\}_{n=1}^{\infty} \]

(b) \[ \{n \sin(3/n)\}_{n=1}^{\infty} \]

(c) \[ \left\{ \frac{e^{4n}}{n^4} \right\}_{n=1}^{\infty} \]
4. **(20 points)** Use a convergence or divergence test to determine whether the following series converge or diverge. Make sure to state what test you are using and show all of your work.

(a) \[ \sum_{n=1}^{\infty} \frac{9n - 1}{5n + 4} \]

(b) \[ \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n + 8} \]
(c) \[
\sum_{n=1}^{\infty} \frac{n \sin(n)}{n^3 + 3}
\]

(d) \[
\sum_{n=1}^{\infty} \frac{2^n}{6^{2n+1}}
\]
5. (18 points)

(a) Compute the area between the functions \( f(x) = x^3 \) and \( f(x) = x^2 \).

(b) Compute the volume of the surface obtained by rotating this region around the \( y \)-axis.
6. (18 points)

(a) Compute the arc length of \( f(x) = 3 + 2x^2 \), \( 0 \leq x \leq 1 \).

(b) Compute the surface area of the solid obtained by rotating the arc \( f(x) = x + 3 \), \( 0 \leq x \leq 1 \) around the \( x \)-axis.
Part B

7. (21 points)

(a) Compute the interval of convergence and the radius of convergence for $\sum_{n=0}^{\infty} \frac{1}{6^n} (x - 3)^n$.

(b) Compute the interval of convergence and the radius of convergence for $\sum_{n=1}^{\infty} \frac{3^n}{n^5} x^n$. 
(c) Compute the radius of convergence for \( \sum_{n=1}^{\infty} \frac{n^n}{n!} x^n \).
8. (20 points) Let \( f(x) = 3 \sin x \), \( g(x) = 2 + \cos x \), and \( q(x) = \frac{f(x)}{g(x)} \).

(a) Write the respective Maclaurin series for \( f(x) \) and \( g(x) \). You may either write the full series using \( \Sigma \) summation notation, or the first three nonzero terms of the series.

(b) Use your answers from part (a) to write the fifth degree Taylor polynomial centered at 0 for \( q(x) \).
(c) Use your answer from part (b) to estimate \( q \left( \frac{1}{10} \right) \).

(d) Use your answer from part (b) and the fact that \( q(x) \) is equal to its Maclaurin series expansion to compute exactly \( q^{(5)}(0) \).
9. (21 points)

(a) Write down the Maclaurin series for the function \( f(x) = \frac{1}{1+x^2} \) using \( \Sigma \) summation notation. What is its radius of convergence?

(b) Write down the Maclaurin series for the function \( F(x) = \arctan x \) using \( \Sigma \) summation notation. What is its radius of convergence? (Hint: If you do not remember the Maclaurin series for \( \arctan x \), your answer from part (a) may be helpful)

(c) Use your answer from part (b) to compute \( \sum_{n=0}^{\infty} \frac{6}{(2n+1)\sqrt{3}} \left( -\frac{1}{3} \right)^n \).
10. **(20 points)** Consider the parametric curve defined by

\[ x = e^t \cos(t), \; y = e^t \sin(t), \; 0 \leq t \leq \pi. \]

(a) Find \( \frac{dy}{dx} \) as a function of \( t \).

(b) Find all points \((x, y)\) on the curve at which the tangent line is horizontal.
(c) Find all points \((x, y)\) on the curve at which the tangent line is vertical.

(d) Find the arc length of the curve.
11. (6 points)

For each of the following points, convert \((x, y)\) to polar coordinates \((r, \theta)\):

(a) \((x, y) = (3, -3)\) \quad r = \quad \theta = 

(b) \((x, y) = (-2, 0)\) \quad r = \quad \theta = 

(c) \((x, y) = (-\sqrt{3}, -1)\) \quad r = \quad \theta = 

For each of the following points, convert \((r, \theta)\) to Cartesian coordinates \((x, y)\):

(d) \((r, \theta) = (5, 3\pi/2)\) \quad (x, y) = 

(e) \((r, \theta) = (4, 7\pi/4)\) \quad (x, y) = 

(f) \((r, \theta) = (3, -11\pi/6)\) \quad (x, y) = 
12. (12 points) Find the area enclosed by one petal of the curve $r = 6 \cos(3\theta)$.
(Note: It may help to sketch the curve)
Trig Identities

- \( \sin^2 \theta + \cos^2 \theta = 1 \)
- \( \tan^2 \theta + 1 = \sec^2 \theta \)
- \( \cot^2 \theta + 1 = \csc^2 \theta \)
- \( \sin(2\theta) = 2 \sin \theta \cos \theta \)
- \( \sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta)) \)
- \( \cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta)) \)
- \( \sin(a + b) = \sin(a) \cos(b) + \cos(a) \sin(b) \)
- \( \sin(a - b) = \sin(a) \cos(b) - \cos(a) \sin(b) \)
- \( \cos(a + b) = \cos(a) \cos(b) - \sin(a) \sin(b) \)
- \( \cos(a - b) = \cos(a) \cos(b) + \sin(a) \sin(b) \)
- \( \sin(a) \cos(b) = \frac{1}{2} [\sin(a - b) + \sin(a + b)] \)
- \( \sin(a) \sin(b) = \frac{1}{2} [\cos(a - b) - \cos(a + b)] \)
- \( \cos(a) \cos(b) = \frac{1}{2} [\cos(a - b) + \cos(a + b)] \)