

EXTRA CREDIT PROBLEM 2

DEADLINE: 5PM FRIDAY SEPTEMBER 25

Define a *isosceles right tetrahedron with side  $s$*  to be the solid region bounded by the four planes defined by the equations

$$x = 0, \quad y = 0, \quad z = 0, \quad \text{and} \quad x + y + z = s.$$

It is the 3-dimensional analog of an isosceles right triangle. Three of its edges have length  $s$  and are perpendicular to each other. The others have length  $s\sqrt{2}$ . We know that its volume is  $s^3/6$ .

Find the volumes of a regular tetrahedron, a regular octahedron and a regular cuboctahedron (look it up), each having edges of length  $s$ . Here are some hints that may help you.

- A regular tetrahedron can be obtained from a regular cube by removing four isosceles right tetrahedra.
- A regular octahedron can be obtained from a regular tetrahedron by removing four smaller regular tetrahedra.
- A regular cuboctahedron can be obtained from a regular cube by removing eight isosceles right tetrahedra.

Show that a regular cuboctahedron is the disjoint (or nonoverlapping) union of eight regular tetrahedra and a number (to be determined by you) of isosceles right tetrahedra.