Math 161 **Spring 2003**

Review - 3

PROBLEMS

1. Evaluate the following limits.

   (a) $\displaystyle\lim_{x\to 0}\frac{\sin x}{x^3}$

   (b) $\displaystyle\lim_{x\to\infty}\frac{\ln(\ln x)}{x}$

   (c) $\displaystyle\lim_{x\to 1}(x-1)\tan\left(\frac{\pi x}{2}\right)$

   (d) $\displaystyle\lim_{x\to 0}\left(\frac{1}{x}-\csc x\right)$

   (e) $\displaystyle\lim_{x\to 0^+}(-\ln x)^x$

2. Let $f(x)=\dfrac{x}{(1+x)^2}$. Find the following:

   (a) domain

   (b) intercepts

   (c) vertacal and horizontal asymptotes

   (d) critical numbers

   (e) intervals of increase and decrease

   (f) local and absolute maxima and minima

   (g) intervals of concavity

   (h) inflection points

   (i) sketch the graph of $f(x)$

3. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side 10 if one side of the rectangle lies on the base of the triangle.

4. Use Newton's method to approximate the root of the equation $x^2-23=0$. Choose a reasonable initial approximation x_1, and use it to find the second approximation x_2.

5. Find $f(x)$ if

   (a) $f'(x)=1-8x^3+2\sin x-\cos x+3e^x,\ f(0)=5.$

   (b) $f''(x)=6-24x^2,\ f'(1)=-3,\ f(2)=-32.$

6. Evaluate the following definite integrals (using the fundamental theorem of calculus or by interpreting the integral in terms of areas)

   (a) $\int_1^3 (3x^2 - 6x + 5)dx$

   (b) $\int_2^8 |x - 4|dx$

   (c) $\int_\pi^{3\pi} \cos x dx$

   (d) $\int_{-1}^{2} \sqrt{4 - s^2}ds$

   (e) $\int_{-e^2}^{-e} \frac{3}{t}dt$

7. Find the derivatives of the following functions

   (a) $f(x) = \int_2^x \sin(t^2)dt$

   (b) $g(x) = \int_{3x}^{5x^2} \sqrt{t}\tan(3t)dt$

8. Estimate the value of $\int_0^{10} (x^2 + 6)dx$ using 5 approximating rectangles and

   (a) left endpoints,

   (b) right endpoints,

   (c) midpoints.

   (d) Evaluate $\int_0^{10} (x^2 + 6)dx$ using the Fundamental Theorem of Calculus.

   (e) Sketch the graph of $f(x) = x^2 + 6$ and explain the meaning of your answers in (a)-(d).

9. Evaluate the following integrals

   (a) $\int x\sin(x^2)dx$

   (b) $\int_0^{\pi/4} \sec^2 x e^{\tan x}dx$

   (c) $\int 3\cot t dt$

   (d) $\int \frac{1}{(2 - 3s)^5}ds$