PRACTICE TEST FOR MATH161 FINAL

HEEKYOUNG HAHN

Part I

(1) Solve the inequality $\ln(x^2 - 2x - 2) \leq 0$.
Answer: $-1 \leq x < 1 - \sqrt{3}$, $1 + \sqrt{3} < x \leq 3$

(2) Evaluate the limit

$$\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}.$$

Answer: $3/2$

(3) Find an equation of the tangent line to the parabola $y = 3x^2 - 5x$ at the point $(2, 2)$.
Answer: $y = 7x - 12$

(4) Find the derivative of $y = \sin^{-1}(2x + 2)$. Simplify where possible.
Answer:

$$y' = \frac{2}{\sqrt{1 - (2x + 2)^2}}$$

(5) If a snowball melts so that its surface area decreases at a rate of $1\,cm^2/min$, find the rate at which the diameter decreases when the diameter is $10cm$.
Answer: $1/(20\pi)\,cm/mim$

Part II

(1)

$$f(x) = 2x^3 - 3x^2 - 12x$$

(a) Find the intervals of increase or decrease.
(b) Find the local maximum and local minimum values.
Answer: (a) increasing on $(-\infty, -1)$ and $(2, \infty)$, decreasing on $(-1, 2)$
Answer: (b) loc. max: 7, loc. min: -20

(2) A right circular cylinder is inscribed in a sphere of radius r.Find the largest possible volume of such a cylinder.
Answer: $V = \frac{4\pi r^3}{3\sqrt{3}}$

(3) Evaluate the limit $\lim_{x\to\infty} x^3 e^{-x}$.
Answer: 0

(4) A car is traveling at $24\, ft/s$ when the brakes are fully applied, producing a constant deceleration of $4\, ft/s^2$. What is the distance covered before car comes to a stop?
Answer: $72\, ft$

(5) Use the fundamental Theorem of Calculus to find the derivative of the function

$$y = \int_{e^x}^{0} \sin^3 t dt.$$

Answer: $-e^x \sin^3(e^x)$

(6) Evaluate the indefinite integral

$$\int \frac{\sin x}{1+\cos^2 x} dx.$$

Answer: $-\tan^{-1}(\cos x) + C$