MATH 161—FINAL EXAM QUESTIONS
December 15, 2004 (4:00–7:00 PM)

1. (8 points) Find the linearization of $f(x) = (x+1)^5$ at $a = 1$ and use it to approximate the number $(2.001)^5$.

2. (8 points) Find the absolute maximum and absolute minimum of $f(x) = x^4 - 2x^2 + 3$ on the interval $[-2, 3]$.

3. (8 points) Prof. Aaron Heap grew up on a farm. One day his dad told him to build a fence around a rectangular field and then divide the field with a fence parallel to one of the sides of the rectangle so that $\frac{1}{3}$ of the field is separated from the rest of the field. Knowing that his son was good at calculus, he simply gave Aaron 180 ft. of fencing material and told him to maximize the area of the larger section of the field. What area did Aaron give the larger section (assuming he didn't make a mistake)?

4. (8 points) A rectangular storage container with no top must have a volume of $20m^3$. The height of the container is 2 times the width of the base. The material for the base costs \$12 per square meter. The material for the sides is \$5 per square meter. Find the cost of the cheapest such container.

5. (10 points) Evaluate the following limits. Use L'Hospital's Rule where appropriate.

(a) $\displaystyle\lim_{x \to -2} \frac{x+2}{x^2+3x+2}$

(b) $\displaystyle\lim_{x \to \infty} \frac{(1+\ell n\, x)^2}{x}$

(c) $\displaystyle\lim_{x \to -\infty} x^2 e^x$

(d) $\displaystyle\lim_{x \to \infty} \left(\sqrt{4x^2+2x} - 2x\right)$

(e) $\displaystyle\lim_{x \to 0^+} (\sin x)^x$

6. (15 points) Let

$$f(x) = \frac{1}{x^2 - 9}\ .$$

(a) On what intervals is f increasing? Decreasing? Find all local maxima and minima of f, if any.

(b) On what intervals is the graph of f concave up? Concave down? Find all points of inflection, if any.

(c) Sketch the graph of f, labeling all points found in parts (a) and (b), and any vertical or hoizontal asymptotes.

7. (15 points) Let

$$f(x) = x^4 + 4x^3\ .$$

(a) On what intervals is f increasing? Decreasing? Find all local maxima and minima of f, if any.

(b) On what intervals is the graph of f concave up? Concave down? Find all points of inflection, if any.

(c) Sketch the graph of f, labeling all points found in parts (a) and (b), all x- and y-intercepts, and showing the behavior of f as $x \to \infty$ and $x \to -\infty$ (that is, $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$).

8. (12 points) Evaluate the following definite or indefinite integrals.

(a) $\displaystyle\int_1^4 (x^2 + 2\sqrt{x} - 5)\, dx$

(b) $\displaystyle\int \frac{3}{t^4}\, dt$

(c) $\displaystyle\int \frac{6}{\sqrt{1-t^2}}\, dt$

(d) $\displaystyle\int_{\pi/2}^{\pi} \cos\theta\, d\theta$

(e) If $\displaystyle\int_2^6 f(x)dx = 14.5$ and $\displaystyle\int_5^6 f(x)dx = 3.7$, find $\displaystyle\int_2^5 f(x)dx$.

(f) Evaluate $\displaystyle\int_{-1}^{2} |x| dx$ by interpreting it in terms of areas. In other words, draw a picture of the region the integral represents, and find the area using high school geometry.

9. (8 points)

(a) Find all numbers $c \in (0, 2)$ for which

$$g(x) = x^3 + x - 1$$

satisfies the Mean Value Theorem.

(b) Differentiate the following functions.

(i) $h(x) = \int_0^{x^3} \sqrt{1 + 2t}\, dt$

(ii) $F(x) = \int_{\sin x}^{2} (e^{3t} + 1) dt$

10. (8 points) Evaluate the following definite or indefinite integrals.

(a) $\int e^{\sin\theta} \cos\theta\, d\theta$

(b) $\int_e^{e^4} \frac{dx}{x\sqrt{\ell n\, x}}$

(c) $\int_0^1 x^2(x^3 + 2)^3\, dx$

(d) $\int_1^2 \frac{e^{1/x}}{x^2}\, dx$