161s07	Sample Midterm 1	Exam Time: , 8:00 - 9:30
Name:		Student No.:

Instructions:

- Answer ALL questions from Section A
- You may use a handwritten sheet of notes. Calculators are NOT permitted.
- Read all questions carefully
- Unless explicitly told otherwise, you should explain all your answers fully.
- Do NOT seperate the pages of your exam.

Problem	Points	Score
A1	8	
A2	8	
A3	8	
A4	8	
A5	9	
A6	9	
A7	9	
A8	9	
Total	68	

Name:

Section A: Answer ALL questions.

Problem A1: [8 pts]

(**a**) Solve $2^{2x} = 4^{3-x}$ for x.

Solution:

$4 = 2^2$ so the equation becomes $2^{2x} = 2^{6-2x}$. Using the fact that the exponential functions are 1-1, this implies that $2x = 6 - 2x$ so $x = 6/4 = 3/2$

(**b**) Describe the following set as a union of intervals: $\{x \big| |x-3| \geq 1\}$

Solution:

$|x-3|$ represents the distance of x from 3. For this to be ≥ 1, x must either be to the right of 4 (including 4)) or be left of 2 (including 2). The set is therefore equal to $(-\infty, 2] \cup [4, \infty)$

(**c**) Simplify the following. Your answer should be an integer: $2\log_2 6 - \log_2 9$

Solution:

Using the log laws

$$2\log_2 6 - \log_2 9 = 2\log_2 6 - 2\log_2 3 = 2(\log_2 \frac{6}{3}) = 2\log_2 2 = 2.$$

(There are other equally valid ways to do this)

Name:

Problem A2: [8 pts] Suppose $f(x) = \sqrt{x}$ and $g(x) = \ln x$. Find

(**a**) The domain of $f \circ g$

Solution:

The domain of f is $[0, \infty)$ and the domain of g is $(0, \infty)$. Thus the domain of $f \circ g$ consists of points x in $(0, \infty)$ such that $\ln x \geq 0$. Now $e^0 = 1$ so $\ln 1 = 0$. The logarithm is increasing so the domain is $(1, \infty)$.

(**b**) The domain of $g \circ f$

Solution:

The domain of $g \circ f$ will consist of points x in $[0, \infty]$ such that $\sqrt{x} > 0$, i.e. $(0, \infty)$.

Name:

Problem A3: [8 pts] The half-life of the metal madeupium is 4-years. Initially a deposit of the metal contains $3kg$ of radioactive metal.
(**a**) The amount of radioactive metal left in the deposit can be written as $m(t) = m_0 e^{kt}$ where k is measured in years. Find m_0 and k.

Solution:

$m_0 = m(0) = 3$ is the original quantity. Now after every 4 years the quantity must halve so $e^{4k} = \frac{1}{2}$, so $k = \frac{1}{4}\ln\frac{1}{2} = -\frac{1}{4}\ln 2$.

(**b**) How long does it take for the amount of radioactive metal remaining to drop to 1kg?

Solution:

$m(t) = 1$ can now be written as $3e^{-\frac{t}{4}\ln 2} = 1$, i.e.

$$-\frac{t}{4}\ln 2 = \ln\frac{1}{3} = -\ln 3$$

so

$$t = 4\frac{\ln 3}{\ln 2}.$$

Name:

Problem A4: [8 pts] For what values of c is the function

$$f(x) = \begin{cases} x^2 - 2, & x \geq c \\ 1 + 2x, & x < c \end{cases}$$

continuous everywhere?

Solution:

The only possible spot for discontinuity is at $x = c$. Now $f(c) = c^2 - 2$

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^-} (1 + 2x) = 1 + 2c$$

$$\lim_{x \to c^+} f(x) = \lim_{x \to c^+} (x^2 - 4) = c^2 - 2$$

so the function is continuous everywhere if $1 + 2c = c^2 - 2$. This means we need c to solve the quadratic equation

$$c^2 - 2c - 3 = (c - 3)(c + 1) = 0$$

which has solution $c = -1$ and 3.

Name:

Problem A5: [9 pts] Find the following limits. If it doesn't exist say DNE. If it is infinite say whether its $+\infty$ or $-\infty$. Carefully explain your reasoning.

(**a**) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^2-6}}{2x+3}$

Solution:

Divide top and bottom by x, since $x \to \infty$ we can assume x is positive so $x = \sqrt{x^2}$. Thus

$$\lim_{x\to\infty} \frac{\sqrt{x^2-6}}{2x+3} = \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x^2}}\sqrt{x^2-6}}{2+\frac{3}{x}} = \lim_{x\to\infty} \frac{\sqrt{1-\frac{6}{x^2}}}{2+\frac{3}{x}} = \frac{1}{2}$$

(**b**) $\lim\limits_{x\to 0^+} \left(\sin x \sin\left(\frac{1}{x}\right)\right)$

Solution:

Now $-1 \le \sin\left(\frac{1}{x}\right) \le 1$ and for small positive x, $\sin x > 0$ so

$$-\sin x \le \sin x \sin\left(\frac{1}{x}\right) \le \sin x$$

Now $\lim\limits_{x\to 0^+} \sin x = 0$ so by the squeeze theorem the limit is 0.

(**c**) $\lim\limits_{x\to 0^-} \dfrac{1}{1-e^x}$

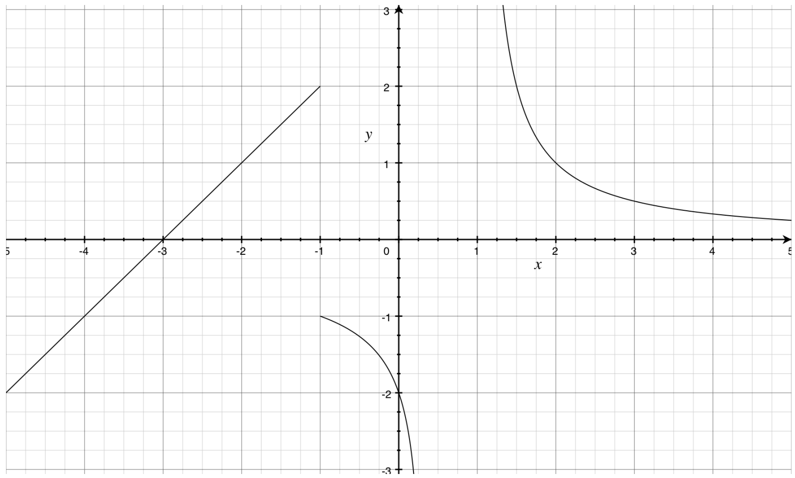
Solution:

Now e^x is continuous and $e^0 = 1$. For x smaller than 0, e^x is smaller than 1. So

$$\lim_{x\to 0^-} \frac{1}{1-e^x} = \lim_{x\to 1^-} \frac{1}{1-x} = +\infty$$

Name:

Problem A6: [9 pts] Consider the following graph of the function $f(x)$.



Find the following limits. If it doesn't exist say DNE. If it is infinite say whether its $+\infty$ or $-\infty$.

(**a**) $\lim_{x\to -1^-} f(x)$

Solution:

2

(**b**) $\lim_{x\to 1^+} f(x)$

Solution:

$+\infty$

(**c**) $\lim_{x\to -2} f(x)$

Solution:

1

(**d**) $\lim_{x\to -1} f(x)$

Solution:

DNE

(**e**) At what points does $f(x)$ fail to be continuous?

Solution:

$x = -1, 1$

(**f**) At what points does $f(x)$ fail to be differentiable?

Solution:

$x = -1, 1$

Name:

Problem A7: [9 pts] Find the tangent line to $y = 3x^2 - x$ at the point $(1, 2)$. (If you compute a derivative, you must use the limit definition)

Solution:

The slope of the tangent line at (1,2) is

$$\lim_{h\to 0} \frac{3(1+h)^2 - (1+h) - 2}{h} = \lim_{h\to 0} \frac{6h + 3h^2}{h} = \lim_{h\to 0} 6 + 3h = 6$$

Therefore the tangent line is

$$y - 2 = 6(x - 1)$$

or

$$y = 6x - 4$$

Name:

Problem A8: [9 pts] The position of a snail at time t is given by $s(t) = \frac{2t}{3+t}$.

(**a**) Where will the snail be after a very long time?

Solution:

This is

$$\lim_{t\to\infty} \frac{2t}{3+t} = \lim_{t\to\infty} \frac{2}{\frac{3}{t}+1} = 2$$

(**b**) Find the velocity function of the snail. (You must use the limit definition of derivative)

Solution:

$$\begin{aligned} v(t) &= \lim_{h\to 0} \frac{\frac{2(t+h)}{3+t+h} - \frac{2t}{3+t}}{h} \\ h &= \lim_{h\to 0} \frac{2(t+h)(3+t) - 2t(3+t+h)}{(3+t)(3+t+h)h} \\ &= \lim_{h\to 0} \frac{6t + 6h + 2t^2 + 2ht - 6t - 2t^2 - 2ht}{(3+t)(3+t+h)h} \\ &= \lim_{h\to 0} \frac{6}{(3+t)(3+t+h)} \\ &= \frac{6}{(3+t)^2} \end{aligned}$$