

**MATH 161 - Calculus IA**  
**MIDTERM EXAM 1**  
**October 24, 2006**

NAME: \_\_\_\_\_

U.R. ID Number: \_\_\_\_\_

Circle your instructor's name and lecture time.

Shannon Starr    TuTh 2:00 - 3:15 PM

Sema Salur        MWF 9:00 - 9:50 AM

Aaron Heap        MWF 10:00 - 10:50 AM

Vijay Sookdeo    MWF 10:00 - 10:50 AM

- No calculators, notes, or books are allowed on this exam.
- Please show all your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Clearly label and circle your final answers.

QUESTION	VALUE	SCORE
1.	9	
2.	8	
3.	9	
4.	9	
5.	10	
6.	15	
7.	16	
8.	9	
9.	15	
TOTAL	100	

**1. (10 points)** Solve the following:

(a)  $|4 - 2x| \geq 3$

(b)  $\ln(x^2 + 1) = 0$

(c)  $2^x = 7^{x+1}$

**2. (8 points)** Let  $f(x) = \ln x$  and  $g(x) = x + 2$ .

(a) Find  $(f \circ g)(x)$  and give its domain.

(b) Find  $(g \circ f)(x)$  and give its domain.

**3. (9 points)** Consider the function  $f(x) = 2 + e^x$ .

(a) Make a rough sketch of  $f(x)$ , and clearly mark any intercepts and asymptotes.

(b) Find domain and range of  $f(x)$ .

(c) Find a formula for the inverse function  $f^{-1}$ , and sketch the graph of  $y = f^{-1}(x)$ , clearly marking any intercepts and asymptotes.

**4. (9 points)** Suppose a colony of cholera bacteria initially contains 100 bacteria and doubles every half hour. Then the population after  $t$  minutes is given by  $p(t) = 100(2)^{kt}$ , for some constant  $k$ .

(a) Find the value  $k$ .

(b) Find the size of the population after 90 minutes.

(c) How long will it take the colony to reach a population of 1000 bacteria?

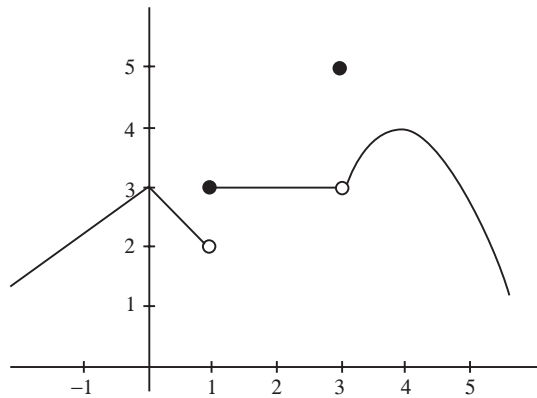
**5. (10 points)**

(a) Define continuity of  $f(x)$  at a point by completing the following sentence. “A function  $f$  is continuous at a point  $a$  if ...”

(b) Find the value of  $b$  so that the following function is continuous at  $x = 2$ .

$$f(x) = \begin{cases} \frac{x^2-5x+6}{x-2} & \text{if } x \neq 2 \\ b & \text{if } x = 2 . \end{cases}$$

6. (15 points) The graph of  $y = f(x)$  is given below:



(a) Calculate the following limits. For those that do not exist, write DNE.

(i)  $\lim_{x \rightarrow 1^-} f(x)$

(ii)  $\lim_{x \rightarrow 1^+} f(x)$

(iii)  $\lim_{x \rightarrow 1} f(x)$

(iv)  $\lim_{x \rightarrow 3^-} f(x)$

(v)  $\lim_{x \rightarrow 3^+} f(x)$

(vi)  $\lim_{x \rightarrow 3} f(x)$

(b) At which values of  $x$  is  $f(x)$  *not* continuous?

(c) At which values of  $x$  is  $f(x)$  *not* differentiable?

**7. (16 points)** Evaluate the following limits if they exist. If a limit does not exist, state whether it is  $+\infty$ ,  $-\infty$ , or neither. Show all work to receive full credit.

(a)  $\lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{x - 2}$

(b)  $\lim_{x \rightarrow 0} \frac{|x|}{x}$

(c)  $\lim_{x \rightarrow -\infty} \frac{2x^2 + 4}{9x^3 - x - 3}$

(d)  $\lim_{x \rightarrow \infty} e^{-x^2}$

**8. (9 points)**  $f(x) = \frac{x^2 + x - 2}{x^2 - 1}$

(a) Evaluate the following limits if they exist. If a limit does not exist, state whether it is  $+\infty$ ,  $-\infty$ , or neither. Show all work to receive full credit.

(i)  $\lim_{x \rightarrow 1^-} f(x)$

(ii)  $\lim_{x \rightarrow 1^+} f(x)$

(iii)  $\lim_{x \rightarrow -1^-} f(x)$

(iv)  $\lim_{x \rightarrow -1^+} f(x)$

(v)  $\lim_{x \rightarrow \infty} f(x)$

(vi)  $\lim_{x \rightarrow -\infty} f(x)$

(b) Find all vertical and horizontal asymptotes of  $f(x)$ .

**9. (15 points)**

(a) What is the definition of  $f'(x)$ ?

(b) Use the definition of derivative to calculate  $f'(x)$  when  $f(x) = x^2 + 2$ .

(c) Find the equation of the tangent line to the curve  $y = x^2 + 2$  at  $x = 3$ .