Name: ________________________________

• Justify your answers.

• No calculators are allowed on this exam, but you are allowed one sheet of paper with writing on both sides.

• The symbol $\mathbb{R}$ stands for the set of real numbers, and $\mathbb{Z}$ stands for the set of integers.

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1. (20 points) Consider the following sets: \( A = \{1, 4, 9, 16\}, \ B = \{-2, -1, 0, 1, 2\}, \ C = \{1, 1, 2, 2, 2, 4\} \).

a) Compute \( A - C \).

b) Compute \( (A \cup C) \cap B \).

c) Compute \(|C|\).

d) Compute the power set \( \mathcal{P}(C) \).
2. (10 points) Is \((p \rightarrow r) \land (q \rightarrow r)\) logically equivalent to \((p \land q) \rightarrow r\)?
3. **(20 points)** Explain whether the following arguments are valid.

a) Every positive real number is the square of a real number.
-1 is not a positive real number.
Therefore, -1 is not the square of a real number.

b) If an object is either a plant or an animal, then it is a living thing.
A rock is not a living thing.
Therefore, a rock is not a plant.
4. (40 points) Consider the following quantified statements, for which the domain of discourse is $\mathbb{R}$. State whether each is true or false. Explain your answer in 1-2 sentences (you do not need to write a formal proof).

a) $\exists x \forall y, xy = 0$

b) $\forall x \exists y, x + y = y$

c) $\forall x \forall y, x + y = y + x$

d) $\exists x \exists y, (x + 2y = 1) \land (2x + 4y = 3)$
5. (40 points) Let $n \in \mathbb{Z}$. For this problem, we say that:

$n$ is type 0 if there exists an integer $k$ such that $n = 3k$,

$n$ is type 1 if there exists an integer $k$ such that $n = 3k + 1$, and

$n$ is type 2 if there exists an integer $k$ such that $n = 3k + 2$.

Every integer is either type 0, type 1, or type 2 (you may assume this fact without proof). Prove the following statements, for which the domain of discourse is $\mathbb{Z}$.

a) If $x$ is type 1 and $y$ is type 2, then $x + y$ is type 0.

b) If $x$ is type 1 and $y$ is type 2, then $xy$ is type 2.
c) No integer is both type 1 and type 2.

d) If $n = x^2$, then $n$ is either type 0 or type 1.
6. (30 points) Consider the functions below. For each one, determine if it is injective and if it is surjective. Prove your assertions.

a) $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = 2x + 3$

b) $g : \mathbb{R} \to \mathbb{R}$ given by $g(x) = 2x + 3$

c) $h : \mathbb{R} \to \mathbb{Z}$ given by $h(x) = \lfloor x \rfloor$