

7. [12] Find all antiderivatives of

$$\frac{1}{(x+1)(x-1)}.$$

You may use the fact that

$$\frac{1}{(x+1)(x-1)} = \frac{1}{2(x-1)} - \frac{1}{2(x+1)}.$$

$$u = x-1 \\ du = dx$$

$$\frac{1}{2} \int \frac{dx}{x-1} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln|u| = \frac{1}{2} \ln|x-1| + C$$

$$u = x+1 \quad \frac{1}{2} \int \frac{dx}{x+1} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln|u| = \frac{1}{2} \ln|x+1| + C$$

$$\text{so } \int \frac{1}{(x+1)(x-1)} = \frac{1}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + C$$

Note this integral may take different forms which are still correct because of the rules of logarithms

8. [10] Evaluate $\int_0^1 e^{e^x} e^x dx$.

$$u = e^x \quad du = e^x dx$$

$$\int_0^1 e^{e^x} e^x dx = \int_{e^0}^{e^1} e^u du = e^u \Big|_{e^0}^{e^1} = e^e - e^1 = e^e - e$$