

MATH 140A

Solutions for Midterm Exam II

November 4, 2003

1. Solve the inequality $\frac{2x+6}{x-2} < 0$.

Notice that $2x+6 = 0$ when $x = -3$, and $x-2 = 0$ when $x = 2$.
 These two numbers split up our numberline into intervals:

	$(-\infty, -3)$	$(-3, 2)$	$(2, /infy)$
$(2x + 6)$	-	+	+
$(x - 2)$	-	-	+
$\frac{2x+6}{x-2}$	+	-	+

So we see that $\frac{2x+6}{x-2} < 0$ when x is in the interval $(-3, 2)$. Notice that we don't include the endpoint -3 because it is strictly less than, and we don't include 2 because then we would have division by zero.

2. Find an equation of the line that goes through the point $(-1, 2)$ and that is perpendicular to $15x - 4y + 16 = 0$.

We can rewrite $15x - 4y + 16 = 0$ as $y = \frac{15}{4}x + 4$. We see that the slope of $15x - 4y + 16 = 0$ is $\frac{15}{4}$ and so a line perpendicular to it will have slope $\frac{-4}{15}$. Putting this into the slope-point formula for a straight line gives: $y - 2 = \frac{-4}{15}(x + 1)$.

3. Find an equation of the circle that has the points $(1, 8)$ and $(5, -6)$ as the endpoints of a diameter.

See page 97 Example 9b in SRW.

4. Let $f(x) = \frac{3+x}{3-x}$.

(a) Find the domain of f .

$$(-\infty, 3) \cup (3, \infty)$$

(b) Find $f(6)$.

$$f(6) = \frac{3+6}{3-6}$$

(c) Find $f(a+h)$.

$$f(a+h) = \frac{3+(a+h)}{3-(a+h)}$$

5. Let $f(x) = \sqrt{x}$. Sketch $f(x)$. Find the domain and range of f .
 See page 164 in SRW for the sketch. The domain=range= $[0, \infty)$.

6. Find an equation for the graph of $f(x) = \frac{1}{x^2}$ that has been shifted down 4 units, right 3 units, then stretched vertically by a factor of 2.

The new equation is $2\left(\frac{1}{(x-3)^2} - 4\right)$

7. Let $f(x) = \sqrt{x-3}$ and $g(x) = x^2 + 2$.

- (a) Find $(f \circ g)(x)$ and its domain.

$$f(g(x)) = f(x^2 + 2) = \sqrt{x^2 + 2 - 3} = \sqrt{x^2 - 1}$$

$$\text{domain } (f \circ g)(x) = (-\infty, -1] \cup [1, \infty)$$

- (b) Find $(g \circ f)(x)$ and its domain.

$$g(f(x)) = g(\sqrt{x-3}) = (\sqrt{x-3})^2 + 2 = x - 1$$

$$\text{domain } (g \circ f)(x) = [3, \infty)$$

8. Let $f(x) = \frac{1}{x}$.

- (a) Find the domain and range of f .

$$\text{domain of } f = (-\infty, 0) \cup (0, \infty)$$

$$\text{range of } f = (-\infty, 0) \cup (0, \infty)$$

- (b) Find $f^{-1}(x)$.

$$\text{Set } y = f(x): y = \frac{1}{x}$$

$$\text{Interchange } x \text{ and } y: x = \frac{1}{y}$$

$$\text{Solve for } y: xy = 1$$

$$y = \frac{1}{x}$$

$$\text{So, } f^{-1}(x) = \frac{1}{x}$$

- (c) Find the domain and range of f^{-1} .

$$\text{domain of } f^{-1} = (-\infty, 0) \cup (0, \infty)$$

$$\text{range of } f^{-1} = (-\infty, 0) \cup (0, \infty)$$

- (d) Check that $(f^{-1} \circ f)(x) = x$.

$$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$$