

MATH 140A

Solutions to Midterm Exam I

1. Use the properties of real numbers to write

$$(5a^4b^{-3}c^7)^2(a^{-5}b^8c^6)^3$$

without any parentheses.

$$\begin{aligned} & (5a^4b^{-3}c^7)^2(a^{-5}b^8c^6)^3 \\ &= 5^2a^8b^{-6}c^{14}a^{-15}b^{24}c^{18} \\ &= 25a^{-7}b^{18}c^{32} \end{aligned}$$

2. Factor $2x^3 - 32x$ completely.

$$2x^3 - 32x = 2x(x^2 - 16) = 2x(x + 4)(x - 4)$$

3. Rationalize the numerator and simplify.

$$\frac{\sqrt{9+x}-3}{x}$$

$$\begin{aligned} & \frac{(\sqrt{9+x}-3)(\sqrt{9+x}+3)}{x(\sqrt{9+x}+3)} = \frac{9+x+3\sqrt{9+x}-3\sqrt{9+x}-9}{x(\sqrt{9+x}+3)} \\ &= \frac{9+x-9}{x(\sqrt{9+x}+3)} = \frac{x}{x(\sqrt{9+x}+3)} = \frac{1}{(\sqrt{9+x}+3)} \end{aligned}$$

4. Perform the indicated operation and simplify

$$\frac{2x}{x-3} - \frac{2}{x+1}$$

$$\begin{aligned} & \left(\frac{2x}{x-3}\right)\left(\frac{x+1}{x+1}\right) - \left(\frac{2}{x+1}\right)\left(\frac{x-3}{x-3}\right) \\ &= \frac{2x(x+1) - 2(x-3)}{(x+1)(x-3)} = \frac{2x^2 + 2x - 2x + 6}{(x+1)(x-3)} \\ &= \frac{2x^2 + 6}{(x+1)(x-3)} = \frac{2(x^2 + 3)}{(x+1)(x-3)} \end{aligned}$$

5. Solve the equation $3x^2 - 6x - 2 = 0$ for x .

Recall that when $ax^2 + bx + c = 0$, we know that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

In this problem, $a = 3$, $b = -6$, $c = -2$.

$$\text{So } x = \frac{6 \pm \sqrt{36 - 4(3)(-2)}}{2(3)} = \frac{6 \pm \sqrt{60}}{6}$$

6. Factor the expression $3x^{5/2} - 6x^{3/2} - 24x^{1/2}$ completely.

$$\begin{aligned} 3x^{5/2} - 6x^{3/2} - 24x^{1/2} &= 3x^{1/2}(x^2 - 2x - 8) \\ &= 3x^{1/2}(x-2)(x-4) \end{aligned}$$

7. Solve the equation

$$\frac{x+1}{x-1} = \frac{-5}{x+3} + \frac{8}{x^2+2x-3}$$

Notice that $x^2 + 2x - 3 = (x+3)(x-1)$, and so we see that this is the common denominator. One way to do this is as follows:

$$(x+3)(x-1)\left(\frac{x+1}{x-1}\right) = \left(\frac{-5}{x+3} + \frac{8}{x^2+2x-3}\right)(x+3)(x-1)$$

$$(x + 3)(x + 1) = -5(x - 1) + 8$$

$$x^2 + 4x + 3 = -5x + 5 + 8$$

$$x^2 + 9x - 10 = 0$$

$$(x + 10)(x - 1) = 0$$

$$x = -10 \text{ or } x = 1$$

A quick check shows that $x = -10$ is a solution, but $x = 1$ is not because it introduces division by zero. So the only solution to this equation is $x = -10$.

8. Solve the equation $|x^2 - 7x + 2| = 10$. You get two equations: $x^2 - 7x + 2 = 10$ or $x^2 - 7x + 2 = -10$. Let's look at $x^2 - 7x + 2 = 10$ first. We have $x^2 - 7x - 8 = 0$, and this factors as $(x + 1)(x - 8) = 0$, so either $x = -1$ or $x = 8$. A quick check shows that both of these are solutions to the original equation. Now let's look at $x^2 - 7x + 2 = -10$. We have $x^2 - 7x + 12 = 0$, and this factors as $(x - 3)(x - 4) = 0$, so either $x = 3$ or $x = 4$. Again, a quick check shows that both of these are solutions to the original equation.

So we see that $x = -1, x = 3, x = 4, x = 8$ are all the solutions to $|x^2 - 7x + 2| = 10$.