

140A f07	Sample Midterm 1	Exam Time: , 8:00 - 9:30
Name:	Student No.:	

**Instructions:**

- Answer ALL questions from Section A
- You may use a handwritten sheet of notes. Calculators are NOT permitted.
- Read all questions carefully
- Unless explicitly told otherwise, you should explain all your answers fully.
- Do NOT separate the pages of your exam.

Problem	Points	Score
A1	10	<input type="text"/>
A2	10	<input type="text"/>
A3	10	<input type="text"/>
A4	15	<input type="text"/>
A5	5	<input type="text"/>
Total	50	<input type="text"/>

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**Section A:** Answer ALL questions.

**Problem A1:** [10 pts] Let  $A = \{x \mid x < 4\}$  and  $B = \{x \mid -1 < x \leq 5\}$ . Express the following sets in **interval** notation.

(a)  $A \cup B$

**Solution:**

Recall that  $\cup$  symbolizes "or" so  $A \cup B$  consists of every point that is either strictly left of 4 OR strictly right of -1 and left of 5. This is everything that is left of 5. So  $\{x \mid x \leq 5\}$  which in interval notation is  $(-\infty, 5]$ .

$$A \cup B = (-\infty, 5]$$

(b)  $A \cap B$

**Solution:**

Recall that  $\cap$  symbolizes "and" so  $A \cap B$  consists of every point that is both strictly left of 4 AND strictly right of -1 and left of 5. This is everything that is strictly left of 4 and strictly right of -1. So  $\{x \mid -1 < x < 4\}$  which in interval notation is  $(-1, 4)$ .

$$A \cap B = (-1, 4)$$

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**Problem A2:** [10 pts] Evaluate the following. Your final answer should be either an integer or a fraction and the simplest form of the solution.

(a)  $|3 - ||2 - |-3|||$

**Solution:**

First note  $|-3| = 3$ , so  $2 - |-3| = 2 - 3 = -1$ . Therefore  $|2 - |-3|| = |-1| = 1$ . The whole thing then equals  $|3 - 1| = |2| = 2$ .

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(b)  $\sqrt{\left(\frac{3}{\sqrt{5}}\right)^{-4}}$

**Solution:**

This is  $\left(\frac{3}{\sqrt{5}}\right)^{-2} = \frac{5^1}{3^2} = \frac{5}{9}$ .

$\frac{5}{9}$

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**Problem A3:** [10 pts] Simplify the following expressions as much as possible

(a)  $\frac{1}{x+1} + \frac{2}{x^2-1}$

**Solution:**

If we factor  $x^2 - 1$  we get  $(x - 1)(x + 1)$  so both denominators share an  $(x + 1)$  term. Thus we can write the sum as

$$\frac{x-1}{(x+1)(x-1)} + \frac{2}{(x+1)(x-1)} = \frac{x+1}{(x-1)(x+1)} = \frac{1}{x-1}.$$

$$\frac{1}{x-1}$$

(b)  $\left(\frac{x^2y^4}{x^{-4}y}\right)^{-\frac{1}{3}}$  **Solution:**

Using the laws of exponents we see that this is equal to

$$(x^6y^3)^{-\frac{1}{3}} = \frac{1}{x^2y}$$

$$\frac{1}{x^2y}$$

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**Problem A4:** [15 pts] Find ALL solutions to the following equations: (note this means that you will not get full credit unless your method guarantees that you have found all solutions)

(a)  $x^2 - 5x + 1 = x - 1$ .

**Solution:**

First put this into standard form as  $x^2 - 6x + 2 = 0$ . We expect two solution since  $(-6)^2 - 4(1)(2) = 28 > 0$ . Since the quadratic seems difficult to factor we shall complete the square. So first we see  $x^2 - 6x = -2$ . This becomes  $(x - 3)^2 - 9 = -2$  and so  $(x - 3)^2 = 7$ . Therefore  $x - 3 = \sqrt{7}$  or  $-\sqrt{7}$ . So  $x = 3 \pm \sqrt{7}$ .

Next we check the solutions. If  $x = 3 + \sqrt{7}$  then  $x^2 - 5x + 1 = (9 + 6\sqrt{7} + 7) - 5(3 + \sqrt{7}) + 1 = 2 + \sqrt{7}$ . Whereas  $x - 1 = 2 + \sqrt{7}$  so that works.

If  $x = 3 - \sqrt{7}$  then  $x^2 - 5x + 1 = (9 - 6\sqrt{7} + 7) - 5(3 - \sqrt{7}) + 1 = 2 - \sqrt{7}$  and  $x - 1 = 2 - \sqrt{7}$ . So this works also.

$$x = 3 \pm \sqrt{7}$$

(b)  $\sqrt{x + 5} = x - 1$

**Solution:**

First square both sides to get  $x + 5 = x^2 - 2x + 1$ . Putting this in standard quadratic form yields  $x^2 - 3x - 4 = 0$ . The quadratic factors as  $(x - 4)(x + 1) = 0$  which has solutions  $x = 4$  and  $x = -1$ .

Now we check. If  $x = 4$  then  $\sqrt{x + 5} = \sqrt{9} = 3$  and  $x - 1 = 3$ . So this one works.

If  $x = -1$  then  $\sqrt{x + 5} = \sqrt{4} = 2$  but  $x - 1 = -2$ . This this one doesn't work.

$$x = 4$$

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(c)  $\frac{1}{\sqrt{x}} + x^{-\frac{3}{2}} = 2.$

**Solution:**

Set  $y = x^{-\frac{1}{2}}$ , then  $y^3 = x^{-\frac{3}{2}}$ . The equation then becomes  $y + y^3 = 2$  or  $y^3 + y - 2 = 0$ . This is a cubic, but we can quickly spot that one solution is  $y = 1$ . We can then factor the cubic as

$$y^3 + y + 2 = (y - 1)(y^2 + By + C)$$

Multiplying out the right hand side yields  $y^3 + (B - 1)y^2 + (C - B)y - C$ . So we must take  $B = 1$  and  $C = -2$ . The equation is then  $(y - 1)(y^2 + y - 2) = 0$ . The quadratic term factors further as  $(y + 2)(y - 1)$  so the equation becomes  $(y - 1)^2(y + 2) = 0$ . The solutions of this are then  $y = 1$  and  $y = -2$ . It just remains to solve for  $x$ . Since  $y = x^{-\frac{1}{2}}$  we have either  $\sqrt{x} = 1$  or  $\sqrt{x} = -\frac{1}{2}$ . Only the first of these has a solution which is  $x = 1$ .

Check: If  $x = 1$ , then  $\frac{1}{\sqrt{x}} + x^{-\frac{3}{2}} = 1 + 1 = 2$ . So yes this works.

(Note. If you had just spotted the  $x = 1$  solution by trial and error, without justifying that there are no other solutions. You would receive very little credit for the problem.)

$x = 1$
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**Problem A5:** [5 pts] Solve the inequality equation  $\frac{1}{2} < \frac{1}{4-2x}$ . Give your final solution in interval notation.

**Solution:**

EITHER Note that the inequality immediately implies that  $4 - 2x > 0$ . Since  $\frac{1}{2} > 0$  also, we can use reciprocals and to change the equation to  $0 < 4 - 2x < 2$  so  $-4 < -2x < -2$  and hence  $2 > x > 1$ . Yielding a solution of  $(1, 2)$ .  
OR change to  $\frac{1}{2} - \frac{1}{2(2-x)} < 0$  so  $\frac{2-x-1}{2(2-x)} < 0$  or simplifying  $\frac{1-x}{2(2-x)} < 0$ .

Now for  $x < 1$  this is  $\frac{+}{+} > 0$ . For  $1 < x < 2$  this is  $\frac{-}{+} < 0$ . For  $x > 2$  this is  $\frac{-}{-} > 0$ . At  $x = 1$  the expression is 0. At  $x = 2$  it is undefined. So the solution is  $(1, 2)$

$(1, 2)$