140A f07	Sample Midterm 2	Exam Time: , 8:00 - 9:30

Name:	Student No.:

Instructions:

- Answer ALL questions from Section A
- You may use a handwritten sheet of notes. Calculators are NOT permitted.
- Read all questions carefully
- Unless explicitly told otherwise, you should explain all your answers fully.
- Do NOT seperate the pages of your exam.

Problem	Points	Score
A1	10	
A2	10	
A3	10	
A4	12	
A5	8	
Total	50	

Name:

Section A: Answer ALL questions.

Problem A1: [10 pts] (**a**) Find the slope-intercept equation for the line L that is perpendicular to $2x-4y-1=0$ and passes through the point $(1,3)$.

Solution:

The slope of the line $2x-4y-1=0$ is $-\frac{2}{-4}=\frac{1}{2}$. Thus if m is the slope of L then $m\left(-\frac{1}{2}\right)=-1$ 0 $m=-2$. A point-slope equation for L is then $y-3=-2(x-1)$. Solving for y, we get the slope-intercept form $y=-2x+5$.

$$y=-2x+5$$

(**b**) Find the equation of the circle for which the line segment from $(1,3)$ to $(5,7)$ is a diameter.

Solution:

The center is the midpoint of line segment which is $\left(\frac{5+1}{2},\frac{7+3}{2}\right)=(3,5)$. The diameter is the distance between the endpoints of the segment, i.e $D=\sqrt{(5-1)^2+(7-3)^2}=\sqrt{32}=4\sqrt{2}$. The radius r is then $D/2=2\sqrt{2}$. The equation for the circle is then $(x-3)^2+(y-5)^2=8$

$$(x-3)^2+(y-5)^2=8$$

Name:

Problem A2: [10 pts] Let $f(x) = \sqrt{4 - x^2}$ and $g(x) = 1 + \frac{3}{x}$.

(**a**) Find $f \circ g(3)$

Solution:

$g(3) = 1 + 3/3 = 2$, so $f \circ g(3) = f(g(3)) = f(2) = \sqrt{4-4} = 0$

$f \circ g\,(0) = \quad 0$

(**b**) Find the simplest form of $g \circ g(x)$ that is valid on its domain.

Solution:

$$g \circ g(x) = 1 + \frac{3}{1 + 3/x} = 1 + \frac{3x}{x+3} = \frac{4x+3}{x+3}.$$

$g \circ g\,(x) = \quad \frac{4x+3}{x+3}$

(**c**) Find the domain of $f \circ g$. Give your answer in interval notation.

Solution:

The domain of g is $(-\infty, 0) \cup (0, \infty)$ and the domain of f is $[-2, 2]$. For x to be in the domain of $f \circ g$ we must therefore have $x \neq 0$ and $-2 \leq g(x) \leq 2$. Thus we must solve $-2 \leq 1 + \frac{3}{x} \leq 2$, $x \neq 0$. But this implies

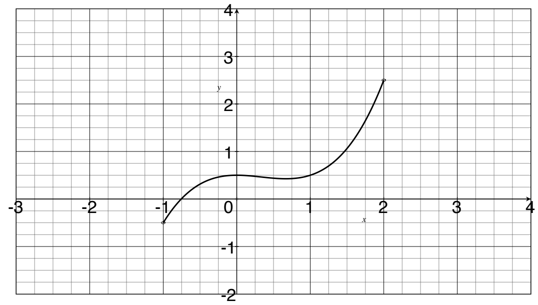
$$-3 \leq \frac{3}{x} \leq 1$$

For $x > 0$, the right inequality implies $x/3 \geq 1$, so $x \geq 3$. For $x < 0$ the left inequality implies $x/3 \leq -1/3$ so $x \leq -1$

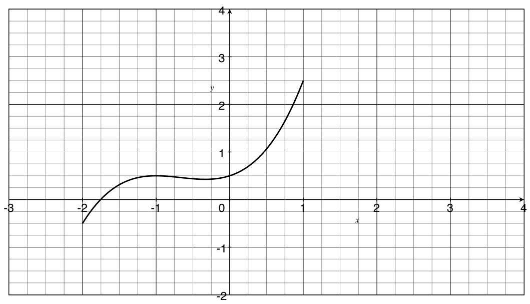
$Dom(f \circ g) = \quad (-\infty, -1] \cup [1, \infty)$

Name:

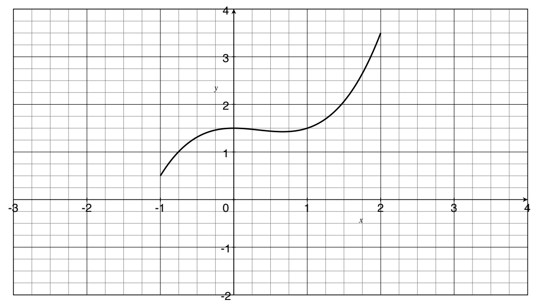
Problem A3: [10 pts] The graph of $y = f(x)$ is shown below. On each of the axes below, sketch the required graph.



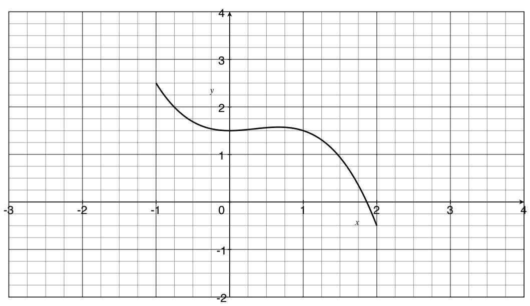
(a) $y = f(x + 1)$



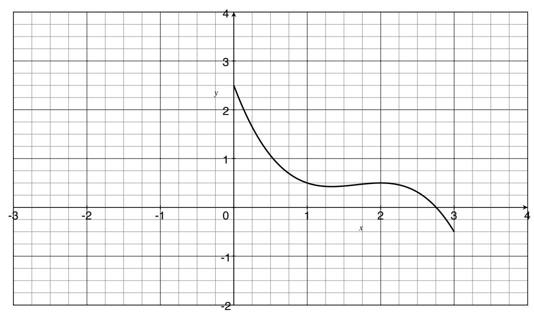
(b) $y = f(x) + 1$



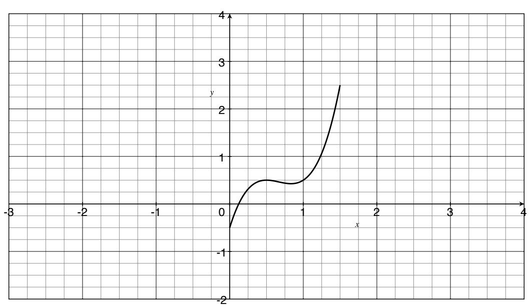
(c) $y = 2 - f(x)$



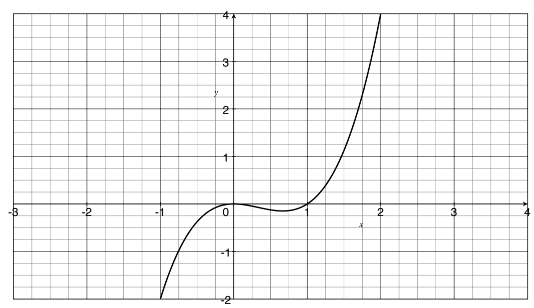
(d) $y = f(2 - x)$



(e) $y = f(2x - 1)$



(f) $y = 2f(x) - 1$



Name:

Problem A4: [12 pts] Let $f(x) = \sqrt{x^2 + 4x + 5}$.

(**a**) If the domain of f is restricted to being the interval$[a, \infty)$, what is the least value of a that ensures f is 1-1?

Solution:

If we complete the square on the quadratic we get $x^2 + 4x + 5 = (x+2)^2 + 1$. We have only take the right half of the quadratic so we insist $x \geq -2$. Since $\sqrt{x}$ is itself 1-1, if we take the domain of f to $[-2, \infty)$, we get that f is 1-1.

$a = \quad -2$

(**b**) If f is restricted to the domain from part (a), find f^{-1}.

Solution:

Set $y = \sqrt{x^2 + 4x + 5}$ then $y^2 = (x+2)^2 + 1$ so $(x+2)^2 = y^2 - 1$. Since the domain of f is $[-2, \infty)$ we must have $x \geq -2$, so we must take the positive square root. Then $x + 2 = \sqrt{y^2 - 1}$ and $x = -2 + \sqrt{y^2 - 1}$.

$f^{-1}(x) = \quad -2 + \sqrt{x^2 - 1}$

Name:

Problem A5: [8 pts] Let $P(x) = (x-2)(x+1)^2$

(**a**) As $x \to \infty$, what happens to $P(x)$?

Solution:

The leading term is $1x^3$, so an odd power with positive coefficient

$P(x) \to \quad \infty$

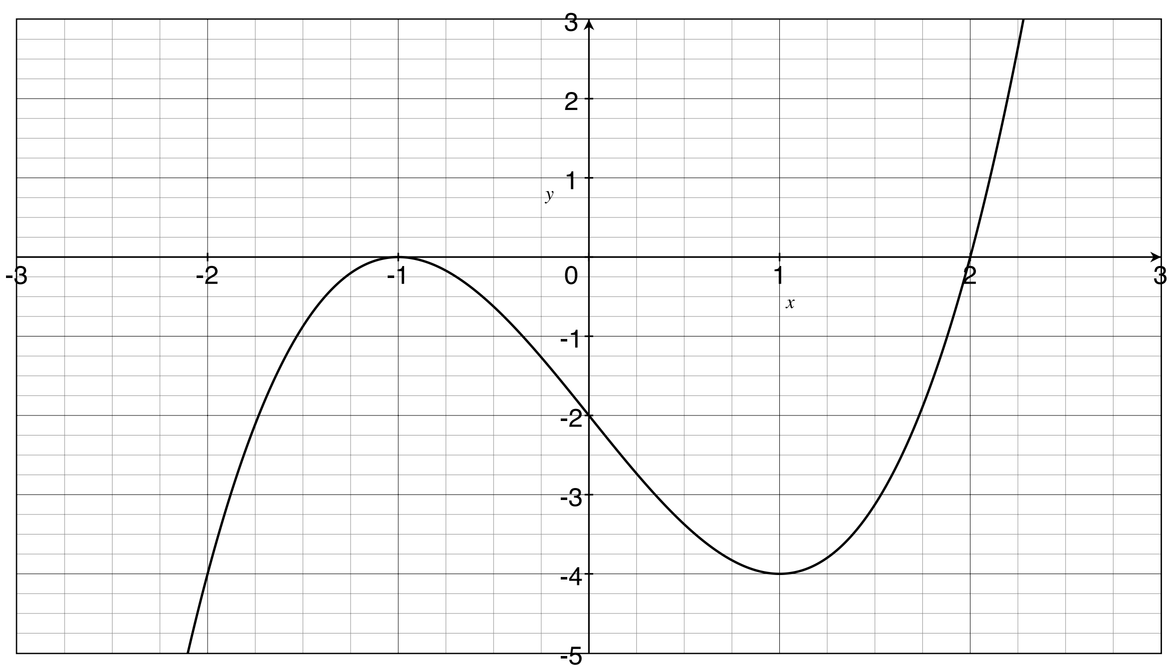
(**b**) As $x \to -\infty$, what happens to $P(x)$?

Solution:

The leading term is again $1x^3$, so an odd power with positive coefficient

$P(x) \to \quad -\infty$

(**c**) Sketch the graph of $y = P(x)$ on the axes below



Solution:

The zeros of $P(x)$ are $x = 2$ and $x = -1$. Near $x = 2$, the graph of $P(x)$ looks like $(2+1)^2(x-2) = 9(x-2)$, i.e. a straight line with slope 9. Near $x = -1$, the graph of $P(x)$ looks like $-3(x+1)^2$, so x^2 reflected vertically, shifted 1 left and stretched vertically by 3. We plot a couple more points $P(0) = -2$ and $P(1) - 4$ to get a better idea of the picture.