UNIVERSITY of ROCHESTER

Math 140A: Calculus with Foundations

Midterm Exam 1

September 30, 2008

Name (please print legibly): ______________________________

University ID Number: ______________________________

Andrew Ledoan	MWF	10:00-11:50 am

- Calculators, cell phones, iPods, and other electronics are not allowed on this exam.
- Please show all your work. You may use the backs of pages if necessary. A correct answer with no work shown will not receive full credit. Please label and circle your final answers.
- You are responsible for checking that this exam has all 6 pages. Please tell us immediately if your exam is missing a page. Missing pages will not contribute to your total score.

Question	Points	Score
1	6	
2	26	
3	30	
4	24	
5	14	
Total:	100	

1. (6 points) Evaluate the expression.

(a) (3 points) $2^{-3} - 3^{-2}$

$$\begin{aligned} 2^{-3} - 3^{-2} &= \frac{1}{2^3} - \frac{1}{3^2} \\ &= \frac{1}{8} - \frac{1}{9} \\ &= \frac{9-8}{72} \\ &= \frac{1}{72} \end{aligned}$$

(b) (3 points) $16^{-3/4}$

$$\begin{aligned} 16^{-3/4} &= (2^4)^{-3/4} \\ &= 2^{(4)(-3/4)} \\ &= 2^{-3} \\ &= \frac{1}{2^3} \\ &= \frac{1}{8} \end{aligned}$$

2. (26 points) Simplify and express without negative exponents.

(a) (5 points) $\dfrac{a^{-2}(b^2c^3)^{-2}}{(a^{-3}b^{-5})^2c}$

$$\begin{aligned}\frac{a^{-2}(b^2c^3)^{-2}}{(a^{-3}b^{-5})^2c} &= \frac{a^{-2}b^{-4}c^{-6}}{a^{-6}b^{-10}c} \\ &= \frac{a^4b^6}{c^7}\end{aligned}$$

(b) (7 points) $\dfrac{x^2+3x+2}{x^2-x-2}$

$$\begin{aligned}\frac{x^2+3x+2}{x^2-x-2} &= \frac{(x+1)(x+2)}{(x+1)(x-2)} \\ &= \frac{x+2}{x-2}\end{aligned}$$

(c) (7 points) $\dfrac{x^2}{x^2-4} - \dfrac{x+1}{x+2}$

$$\begin{aligned}\frac{x^2}{x^2-4} - \frac{x+1}{x+2} &= \frac{x^2}{(x-2)(x+2)} - \frac{x+1}{x+2} \\ &= \frac{x^2}{(x-2)(x+2)} + \frac{-(x+1)(x-2)}{(x-2)(x+2)} \\ &= \frac{x^2-(x^2-x-2)}{(x-2)(x+2)} \\ &= \frac{x+2}{(x-2)(x+2)} \\ &= \frac{1}{x-2}\end{aligned}$$

(d) (7 points) $\dfrac{x^2+x-2}{x^2-6x+9} \div \dfrac{x^2-1}{x-3}$

$$\begin{aligned}\frac{x^2+x-2}{x^2-6x+9} \div \frac{x^2-1}{x-3} &= \frac{x^2+x-2}{x^2-6x+9} \cdot \frac{x-3}{x^2-1} \\ &= \frac{(x-1)(x+2)}{(x-3)^2} \cdot \frac{x-3}{(x-1)(x+1)} \\ &= \frac{x+2}{(x-3)(x+1)}\end{aligned}$$

3. (30 points) Factor the expression completely.

(a) (7 points) $2x^2 + 5x - 12$

$$2x^2 + 5x - 12 = (2x - 3)(x + 4)$$

(b) (7 points) $x^3 - 3x^2 - 4x + 12$

$$\begin{aligned} x^3 - 3x^2 - 4x + 12 &= x^2(x - 3) - 4(x - 3) \\ &= (x - 3)(x^2 - 4) \\ &= (x - 3)(x - 2)(x + 2) \end{aligned}$$

(c) (8 points) $x^4 - 2x^2 - 3$

$$\begin{aligned} x^4 - 2x^2 - 3 &= (x^2 - 3)(x^2 + 1) \\ &= (x - \sqrt{3})(x + \sqrt{3})(x^2 + 1) \end{aligned}$$

(d) (8 points) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$

$$\begin{aligned} 3x^{3/2} - 9x^{1/2} + 6x^{-1/2} &= 3x^{-1/2}(x^2 - 3 + 2) \\ &= 3x^{-1/2}(x - 2)(x - 1) \end{aligned}$$

4. (24 points) Find all real solutions.

(a) (8 points) $2x^2 + 4x + 1 = 0$ Here $a = 2$, $b = 4$, and $c = 1$. Since the discriminant is

$$b^2 - 4ac = 4^2 - 4(2)(1) = 16 - 8 = 8 > 0,$$

the given equation has two distinct real roots. Using the quadratic formula, we obtain

$$x = \frac{-4 \pm \sqrt{8}}{2(2)} = \frac{-4 \pm 2\sqrt{2}}{4} = \frac{-2 \pm \sqrt{2}}{2}.$$

(b) (8 points) $\sqrt{5 - x} + 1 = x - 2$

$\sqrt{5-x} + 1 = x - 2 \iff \sqrt{5-x} = x - 1 \Rightarrow 5 - x = (x-3)^2 \iff 5 - x = x^2 - 6x + 9 \iff 0 = x^2 - 5x + 4 = (x-4)(x-1)$. Potential solutions are $x = 4$ and $x = 1$. We must check each potential solution in the original equation.

Checking $x = 4$: LHS = $\sqrt{5 - (4)} + 1 = \sqrt{1} + 1 = 2$; RHS = $(4) - 2 = 2$. Yes.

Checking $x = 1$: LHS = $\sqrt{5 - (1)} + 1 = \sqrt{4} + 1 = 2 + 1 = 3$; RHS = $(1) - 2 = -1$. No.

The only solution is $x = 4$.

(c) (8 points) $3|x - 4| = 10$

$3|x-4| = 10 \iff 3|x-4| = 10 \iff |x-4| = 10/3 \iff x = 4 = \pm 10/3 \iff x = 4 \pm 10/3$. So $x = 4 - 10/3 = 2/3$ or $x = 4 + 10/3 = 22/3$. Thus, the solutions are $x = 2/3$ and $x = 22/3$.

5. (14 points) Solve the inequality

$$\frac{2x-3}{x+1} \leq 1.$$

Express the solution using interval notation and graph the solution set on the real number line.

$\frac{2x-3}{x+1} \leq 1 \iff \frac{2x-3}{x+1} - 1 \leq 0 \iff \frac{2x-3}{x+1} - \frac{x+1}{x+1} \leq 0 \iff \frac{x-4}{x+1} \leq 0$. The expression on the left of the inequality changes sign where $x = -4$ and where $x = 1$. Thus, we must check the intervals in the following table.

Interval	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
Sign of $x - 4$	$-$	$-$	$+$
Sign of $x + 1$	$-$	$+$	$+$
Sign of $\frac{x-2}{x+1}$	$+$	$-$	$+$

Since $x - 1$ makes the expression in the inequality undefined, we exclude this value. The deserive interval is $(-1, 4]$.